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Reconstruction of Quantum Mechanics with Information Operators

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Abstract. We reconstruct quantum mechanics by introducing "information operators" and excluding the concept of wave functions. Multiple information operators simultaneously describe a single system and continuously develop in time even in the process of a measurement. We also introduce the concept of condensation for a system with many degrees of freedom in a rather general meaning. In terms of the multiplicity of description and the condensation, we explain quantum phenomena including measurements without the collapse of the wave function.

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1. Introduction

Quantum mechanics has been developed continuously from the discovery by Heisenberg and Schrödinger. The principles of quantum mechanics are clearly mentioned in the traditional book of Dirac [1]. The development includes improved formalisms and various calculation methods, and produces many applications. Nowadays, by a highly developed experimental technology, it is even possible to observe the interference effect of a single electron, which is proper to quantum mechanics [2]. Further it is now a realistic destination to control and utilize fundamental quantum effects like quantum entanglement.

On the other hand, the interpretation of quantum mechanics has been controversial from the time of discovery. In particular, researchers have argued the physical meaning of a wave function from various standpoints. Among them, the Copenhagen interpretation is the most accepted one. However, there are many people who do not convince themselves that the interpretation explains all the quantum phenomena.

In the interpretation issue, the collapse of the wave function is the most serious one. The wave function describing a physical system seems to change discontinuously from a time before the measurement to a time after it. Following von Neumann [3], if we pursue this problem, it comes at the human consciousness. I think that this argument is inevitable if we consider the problem within the conventional framework of quantum mechanics. In my opinion, von Neumann has not completed the argument, since he did not argue what the collapse of a wave function at the consciousness is in detail.

The existence of a mixed state brings further difficulties to the interpretation of quantum mechanics. While a pure state is represented as a wave function, a mixed state can only be represented as a density operator. Without the mixed state, we cannot describe many quantum phenomena in real experiments. Despite its necessity, there is unclearness in the meaning of the mixed state.

In a typical explanation, the mixed state is not a state, and the system must be strictly in a single pure state described by a state vector, or a wave function. However, if we have not enough knowledge of the state vector, then we express the situation with lacked information as a mixed state described by a density operator. In the explanation, the wave function is fundamental and the density operator is secondarily constructed of wave functions. This idea seems to be reasonable, if we statistically treat an ensemble consisting of many equivalent systems where each system is strictly in one of the possible state vectors. By this interpretation, we can calculate the average of a physical quantity over the ensemble without ambiguity.

The above argument is not applicable, if we treat a single quantum system and perform a single measurement to it. In this situation, there is no ensemble and we do not repeat the same experiment. Hence I doubt the proposition that the density operator is a secondary concept and what is essential is a state vector. Furthermore, the discontinuity corresponding to the collapse of a wave function still remains irrespective of the interpretation of the mixed state. We lack substantial understanding of both the pure state and the mixed state.

In this paper, we reconstruct quantum mechanics on a standpoint totally different from the conventional one. In the reconstructed quantum mechanics, we have no wave functions. Instead, we introduce 'information operators' which carry information on the system. Although an information operator is mathematically similar to a density operator of the conventional quantum mechanics, it is physically different. In particular, there exist multiple information operators which simultaneously describe the same system. When we have an information operator, the reconstructed quantum mechanics gives an information operator at a later time, which may be a time after a measurement. Another essential concept is the condensation of a system. We show that a condensed system plays a role of an apparatus of measurement. By the concepts of the multiple description and the condensation, we consistently explain quantum phenomena including measurements without discontinuity like the collapse of a wave function.

This paper is organized as follows. In section 2, we examine multiple descriptions of a classical dice for an instructive preparation. In section 3, we introduce information operators to describe multiply a quantum system. In section 4, we mention the time-development of the information operator. In section 5, we introduce the concept of condensation for a system with large degrees of freedom. In section 6, we define the composite system. In section 7, we argue the condensation of a composite system. In section 8, we define the measurement and the observable. In section 9, we introduce the information vector, which corresponds to the wave function in the conventional quantum mechanics. In section 10, we demonstrate the reconstructed quantum mechanics with

the interpretation of the Stern-Gerlach experiment. In section 11, we examine an electron passing through the slits in a screen in terms of the reconstructed quantum mechanics. Section 12 is devoted to summary.

2. Multiple Description of a Dice

We treat multiple inequivalent descriptions of a single system in the following sections. To prepare for this unconventional idea, we examine multiple descriptions of a usual dice, although it is a classical object and the analogy is rather restrictive.

When we roll a dice and have the 3-spot, the following four descriptions are all correct:

(i) The dice shows any number of spots; $A = \{1, 2, 3, 4, 5, 6\}$.

(ii) The dice shows odd spots; $B = \{1, 3, 5\}$.

(iii) The dice shows less than or equal to 3 spots; $C = \{1, 2, 3\}$.

(iv) The dice shows just three spots; $D = \{3\}$.

Here each description is followed by the corresponding set. We say, for example in (ii), that the dice is described by B. We also say that A is the maximum description and D is the minimum description of the dice. On the other hand, the following descriptions are incorrect:

(v) The dice shows even spots; $F = \{2, 4, 6\}$.

(iv) The dice shows just one spot; $G = \{1\}$.

Hence the dice is neither described by F nor G.

Now suppose we only have the information that the dice is described by B. In other words, we only know that the dice shows one of the 1, 3 and 5-spots. Then we ask which set describes the dice except for B. For example, the dice is described by A, too, since the cases of the 1, 3 and 5-spots are included in the cases of the 1, 2, 3, 4, 5 and 6-spots. In general, if $X \supset Y$, then the following is correct: if the dice is described by Y, then it is also described by X. Since A is the maximum description, it always describes the dice.

Next, we consider whether the dice is described by D or not, when the dice is described by B. Since the dice may show one of the cases of the 1, 3 and 5-spots, we cannot definitely say that the dice is described by $D = \{3\}$. It might be described by $G = \{1\}$. In general, if $X \supset Y$, then the following is correct: if the dice is described by X, then it is not necessarily described by Y. Instead, we say that the dice is described by D with probability $\frac{1}{3}$, since the possibility is one of the three sets, $\{1\}$, $\{3\}$ and $\{5\}$.

Further, we compare B with C. There is no inclusion relation between them, although both describe the dice with the 3-spot simultaneously. This is an example that two sets simultaneously describe the dice without no inclusion relation between them.

In the quantum mechanics reconstructed below, a system is also multiply described. However the description is crucially different from that of a dice. We have described the dice with the 3-spot by A, B or C because of the lack of information. In contrast,

for a system in the reconstructed quantum mechanics, there is no idea corresponding to the fact that the dice actually shows the 3-spot. What we can ask is only about the information given or obtained. In this way, we will avoid the discontinuity in the observation of the quantum mechanics. Another fundamental difference is that a system is described by operators instead of sets.

3. Information Operators

We reconstruct quantum mechanics starting from the following postulate:

Postulate 1 (Existence of a Hilbert space): *There is a complex Hilbert space $\mathcal{H}_S$ for a system S.*

In this Hilbert space $\mathcal{H}_S$, we define operators which can represent information of the system S, as follows:

Definition 1 (Information operator): *When an Hermitian operator ρ in the Hilbert space $\mathcal{H}_S$ satisfies the condition,*

$$\mathrm{tr}\{\rho\} = 1, \quad \rho \geq 0, \tag{1}$$

it is an information operator, or an i-operator shortly.

When the dimension of $\mathcal{H}_S$ is of a finite value d, we call $\rho^{\max} \equiv \frac{1}{d} I$ the maximum i-operator, where I is the identity operator. If the dimension of $\mathcal{H}_S$ is infinite, there is no maximum i-operator. An i-operator $\rho^{\mathrm{pure}} = |\psi\rangle\langle\psi|$ with any normalized vector $|\psi\rangle$ in $\mathcal{H}_S$ is called a pure i-operator.

Any i-operator is represented as

$$\rho = \sum_i p_i \, |\psi_i\rangle\langle\psi_i| \tag{2}$$

with a set of appropriate normalized vectors $\{|\psi_i\rangle\}$ in $\mathcal{H}_S$, where $p_i \geq 0$ for all i and $\sum_i p_i = 1$.

The following is a postulate about the description of a system.

Postulate 2 (Description of the system): *System S is described by i-operators in $\mathcal{H}_S$.*

This means that we can derive some information from the i-operator about the system S. In other words, we can predict results of a measurement with probabilities which the i-operator gives, as will be argued later. It is an important part for solving a problem to find an i-operator which describes the system under consideration.

Postulate 2 never means that an i-operator uniquely describes the system, and, in general, there are multiple i-operators simultaneously describing the same system. If one knows more than two i-operators to describe a system, one can choose anyone of them as will be explained later.

In the postulates of this paper, we do not require a *state*, which plays a central role in the conventional quantum mechanics. It is an incorrect idea that an i-operator

incompletely describes a system owing to the lack of information. Multiple i-operators describing the same system equally satisfy the same postulates in this paper. In particular there is no absolute i-operator which plays a special role.

Here we mention a definition and a postulate about a special relation between two i-operators.

Definition 2 (Expansion and contraction of an i-operator): *If there exists an operator K, and i-operators ρ_1 and ρ_2 satisfy the following relation,*

$$K\rho_1 K^\dagger = \rho_2, \tag{3}$$

then ρ_1 is an expansion of ρ_2, and ρ_2 is a contraction of ρ_1. K is the contracting operator which contracts ρ_1 to ρ_2.

Postulate 3 (Description of the system by an expansion): *If an i-operator describes a system, any expansion of the i-operator also describes the system.*

As for the reverse, even if an i-operator ρ describes a system, $K\rho K^\dagger$ with any operator K does not necessarily describe the system. We have the following theorems about the multiple descriptions by i-operators.

Theorem 1 (Unitary transformation): *If an i-operator describes a system, then another i-operator unitarily transformed from it also describes the system.*

Proof: In (3), the contracting operator K may be a unitary operator. □

Theorem 2 (Description by the maximum i-operator): *If the dimension of the Hilbert space of a system is finite, the maximum i-operator always describes the system.*

Proof: We show that the maximum i-operator is an expansion of any i-operator. Let $\rho_1 = \frac{1}{d}I$ in (3) so that $KK^\dagger = d\,\rho_2$. For arbitrary ρ_2, we take the representation where ρ_2 is diagonal and its eigenvalues are $\rho_2^{(i)}$ $(i = 1, 2, \cdots, d)$. We take K such that the diagonal elements are $\sqrt{\rho_2^{(i)} d}$ $(i = 1, 2, \cdots, d)$ and all the off-diagonal elements are 0. Then $KK^\dagger = d\,\rho_2$ is satisfied. □

Theorem 3 (Composition of i-operators): *If i-operator ρ is composed of i-operators ρ_i $(i = 1, 2, \cdots)$, i. e.*

$$\rho = \sum_i p_i \rho_i \tag{4}$$

with $\sum_i p_i = 1$ and $p_i > 0$ $(i = 1, 2, \cdots)$, then each ρ_i is a contraction of ρ.

Proof: Let the eigenvalues of ρ be $\rho^{(1)}$, $\rho^{(2)}$, $\cdots$ in ascending order, and the normalized eigenvectors belonging to them be $|\rho^{(1)}\rangle$, $|\rho^{(2)}\rangle$ $\cdots$. Similarly let the eigenvalues of ρ be $\rho_1^{(1)}$, $\rho_1^{(2)}$, $\cdots$ in ascending order, and the normalized eigenvectors belonging to them be $|\rho_1^{(1)}\rangle$, $|\rho_1^{(2)}\rangle$ $\cdots$. Clearly, the number of zero eigenvalues of ρ is smaller than that of ρ_1. Hence there exist non-negative real numbers a_i $(i = 1, 2, \cdots)$ which satisfy $a_i \rho^{(i)} = \rho_1^{(i)}$ $(i = 1, 2, \cdots)$. By taking K as $K \equiv \sum_i \sqrt{a_i} |\rho_1^{(i)}\rangle\langle\rho^{(i)}|$ we have $K\rho K^\dagger = \rho_1$. □

We define the amount of information of an i-operator by the entropy below:

Definition 3 (Entropy): *The amount of information, or the entropy of an i-operator ρ is defined as*

$$E[\rho] = -\mathrm{tr}\{\rho \log \rho\}. \tag{5}$$

In particular, we have $E[\rho^{\max}] = \log d$ and $E[\rho^{\mathrm{pure}}] = 0$. The entropy of a general i-operator is a value between these. It is emphasized that the entropy is defined for each i-operator describing a system and not for the system itself. Different i-operators may have different entropies even for the same system.

Below we mention a postulate about the idea that a system is described by an i-operator with probability.

Postulate 4 (Description with probability): *Let an i-operator ρ be composed of i-operators ρ_1 and ρ_2 as*

$$\rho = p_1\rho_1 + p_2\rho_2 \tag{6}$$

with $p_1 > 0$, $p_2 > 0$ and $p_1 + p_2 = 1$. If ρ describes a system, then ρ_1 describes the system with probability p_1, and ρ_2 does with probability p_2.

In Postulate 4, $\rho_1\rho_2 = 0$ is not required. If we write (6) as $\rho = \sigma_1 + \sigma_2$, then we have the probabilities, $p_1 = \mathrm{tr}\,\sigma_1$ and $p_2 = \mathrm{tr}\,\sigma_2$. By using (6) successively, Postulate 4 is applicable to the case of an i-operator composed of more than two i-operators ρ_i ($i = 1, 2, \cdots$): i. e. if $\rho = \sum_i p_i\rho_i$ describes a system, then the probability that ρ_i describes the system is p_i.

For example, we consider a system which consists only of a spin $\boldsymbol{s} = (s_x, s_y, s_z)$ of magnitude 1. Let the eigenvectors of s_z belonging to eigenvalues $+1$, 0 and -1 be $|+\rangle$, $|0\rangle$ and $|-\rangle$, respectively. When we have information that the present eigenvalue of the system is not zero, we seek an i-operator describing the system. An i-operator describing the system is clearly $\rho' = \frac{1}{2}(|+\rangle\langle+| + |-\rangle\langle-|)$, since the eigenvalue of s_z is not zero, and it is $+1$ or -1 with equal possibility. By Theorem 2, i-operator $\rho^{\max} = \frac{1}{3}(|0\rangle\langle0| + |+\rangle\langle+| + |-\rangle\langle-|)$ also describes the system. There is no contradiction in that multiple i-operators simultaneously describe the same system. ρ' describes the system more definitively than $\rho^{\max}$ does, because $E[\rho'] = \log 3$ and $E[\rho^{\max}] = \log 2$. We next consider i-operators $\rho^+ = |+\rangle\langle+|$ and $\rho^- = |-\rangle\langle-|$ in this case. Both ρ^+ and ρ^- are contractions of ρ' from Theorem 3, since $\rho' = \frac{1}{2}(\rho^+ + \rho^-)$. Then they do not describe the system definitively. From Postulate 4, the system is described by ρ^+ with probability $\frac{1}{2}$, and is described by ρ^- with probability $\frac{1}{2}$.

4. Time Development

We consider the time development of the i-operator as follows.

Postulate 5 (Time development of the i-operator): *Let i-operator $\rho(t)$ describe system S at time t. If the system S is isolated until time t' after t, then there exists a*

unitary operator $U(t,t')$ proper to the system such that the i-operator

$$\rho(t') = U(t,t')\rho(t)\,U^{\dagger}(t,t') \tag{7}$$

describes the system at time t'. $\rho(t)$ is continuous with respect to t.

If we have an i-operator describing a system at time t, we can calculate an i-operator describing it at a later time t' from (7) until the system is isolated. Unitary operator $U(t,t')$ is proper to the system, and any i-operators describing the same system develop by the same $U(t,t')$. $U(t,t')$ is continuous with respect to t and t' since $\rho(t)$ is continuous. As will be mentioned later, a time development with measurement is also continuous.

Under the time uniformity, the unitary operator $U(t,t')$ is written with an Hermitian operator H as

$$U(t,t') = \exp\{-i(t'-t)H/\hbar\}, \tag{8}$$

where $\hbar$ is Plank's constant divided by 2π. Then we call H the Hamiltonian.

From (7) and (8), an i-operator $\rho(t)$ describing the system satisfies

$$i\hbar\frac{d\rho}{dt} = H\rho - \rho H. \tag{9}$$

We call (9) the equation of motion.

5. Condensation and Classical System

We propose the concept of condensation which is expected in a system with many degrees of freedom.

Definition 4 (Condensation of a system): *System T is condensed in the period between τ_1 and τ_2, if the following conditions are satisfied:* (i) *The Hilbert space of the system T is divided into a finite or an infinite number of subspaces, and* (ii) *if an i-operator describing the system belongs to one of the subspaces at a time in this period, then the i-operator belongs to the same subspace during the period.*

We call each of the above subspaces the subspace of condensation, and call the period $\Delta\tau = \tau_2 - \tau_1$ the period of condensation. An i-operator belonging to a subspace of condensation brings 0 if it operates on any vector out of the subspace. If a condensed system is described by an i-operator $\rho = p_1\rho_1 + p_2\rho_2$ with ρ_1 and ρ_2 which belong to different subspaces of condensation, then probabilities p_1 and p_2 do not change in the period ($\tau_1 < t < \tau_2$) of condensation.

A system which is condensed is supposed to be of a large number of degrees of freedom. Especially, in a system with an infinite number of degrees of freedom, the Hilbert space is eternally divided into subspaces of condensation, so that there exists no local operator which transforms a vector in a subspace into a vector in another subspace. This is the case of $\tau_1 \to -\infty$ and $\tau_2 \to +\infty$. This phenomenon corresponds to the spontaneous symmetry breaking in a conventional quantum theory for a system with an infinite number of degrees of freedom. The condensation is a property of the Hamiltonian of the system, or the time-development unitary transformation.

We label a subspace of condensation by m, and denote it as $\mathcal{H}_T^m[\tau_1, \tau_2]$ or simply $\mathcal{H}_T^m$. Then the total Hilbert space is written as $\mathcal{H}_T = \oplus_m \mathcal{H}_T^m$. The label m takes discrete values if the number of the subspaces is countable, and continuous values if it is uncountable.

By defining the projection operator to Hilbert space $\mathcal{H}_T^m$ as P_T^m, we have the identity

$$\rho_T(t) = \sum_m P_T^m \rho_T(t) P_T^m, \quad (\tau_1 < t < \tau_2) \tag{10}$$

where, if m is continuous, the symbol of summation means integration. If the system T is described by $\rho_T(t)$, the probability that the system is described by an i-operator in the subspace with label m, is given as

$$p_T^m = \mathrm{tr}\{P_T^m \rho_T(t) P_T^m\} \quad (\tau_1 < t < \tau_2) \tag{11}$$

from (10) and Postulate 4. The probability p_T^m is constant in the period of $\tau_1 < t < \tau_2$. We can know the label m by a measurement as mentioned later. If we know a value of the label m, we can exclude the possibilities other than m in (10). Then we can also describe the system by only the term with m as

$$\rho_T^m(t) = \frac{P_T^m \rho_T(t) P_T^m}{\mathrm{tr}\{P_T^m \rho_T(t) P_T^m\}}, \quad (\tau_1 < t < \tau_2) \tag{12}$$

where the denominator is introduced for the normalization of $\mathrm{tr}\{\rho_T^m\} = 1$. Since (10) is rewritten as

$$\rho_T(t) = \sum_m p_T^m \rho_T^m(t), \quad (\tau_1 < t < \tau_2) \tag{13}$$

$\rho_T(t)$ is a common expansion of $\rho_T^m(t)$ with arbitrary m. The system is described by $\rho_T^m(t)$ if we include the information of the label m, while it is simultaneously described still by $\rho_T(t)$ which lacks the information.

We define a classical system based on the concept of condensation as follows.

Definition 5 (Classical system): *Let a condensed system be described by an i-operator which belongs to a subspace labeled by m. We observe the value of m in a period fairly longer than the period of condensation. The i-operator changes the belonging subspace slowly and successively in the observed period. If we do not concern ourselves with anything but the time development of the label m, then the system is defined to be a classical system.*

Classical mechanics is supposed to describe the motion or the time development of the label m with deleting or averaging the other degrees of freedom.

Now we refer to a famous subject known as Schrödinger's cat. The typical situation is as follows: Let an alive cat be in an nontransparent box with a lid together with an appropriate amount of radium, a detector of α-particles, and a device scattering hydrocyanic acid by trigger of α-particles. After a while, say Δt, the cat dies owing to hydrocyanic acid if an α-particle is detected, while it lives if not. Let the probabilities of ejecting and not ejecting an α-particle in the period Δt be respectively p^+ and p^- ($p^+ + p^- = 1$).

The cat, which we call system T, is of a large number of degrees of freedom and the problem of "dead or alive" of the cat is explained in terms of condensation. That the cat is alive is that system T is condensed in the subspace with label $m = +$. Similarly that the cat is dead is that system T is condensed in the subspace with label $m = -$. We denote i-operators describing them as ρ_T^+ and ρ_T^-, respectively.

After the period Δt, if one finds whether the cat is dead or alive by looking into the box and includes the fact as information, then the cat is described by only one of ρ_T^+ and ρ_T^- depending on the fact. On the other hand, $\rho_T \equiv p^+\rho_T^+ + p^-\rho_T^-$ is a common expansion of ρ_T^+ and ρ_T^-, and always describes the cat. ρ_T is the i-operator where one did not look into the box or did look but did not include the fact as information. This just means that the probabilities of the cat alive and dead are p^+ and p^-, respectively, when one neglects or does not include the information of the fact about the cat. Hence the double descriptions by ρ_T and one of ρ_T^+ and ρ_T^- are not in contradiction.

Since the cat is condensed for a long time, even a part of ρ_T^+ (ρ_T^-) does not change into an i-operator in the subspace of $m = -$ ($m = +$). Further there is no i-operator corresponding to a vector superposed of vectors in the different subspaces unlike the conventional quantum mechanics.

The reason why Schrödinger's cat is paradoxical in the conventional quantum mechanics is that a condensed system is not distinguished from an non-condensed system and also a multiple description of a system is not considered.

6. Composite System

Consider two systems S and T, whose Hilbert spaces are $\mathcal{H}_S$ and $\mathcal{H}_T$, respectively. We denote the composite system consisting of S and T by $S + T$, and the Hilbert space by $\mathcal{H}_{S+T}$=$\mathcal{H}_S \otimes \mathcal{H}_T$. We require the following postulate for consistency.

Postulate 6 (i-operator for a composite system): *If i-operators ρ_S and ρ_T describe systems S and T, respectively, then the tensor product $\rho_S \otimes \rho_T$ is an i-operator which describes the composite system $S + T$.*

If an i-operator is of the tensor product form, $\rho_S \otimes \rho_T$, then it is called to be separated into ρ_S and ρ_T. We further require the following postulate about the separation of i-operators.

Postulate 7 (Separation of an i-operator): *If systems S and T does not interact with each other, and an i-operator ρ_T describes the system T, then there exists at least one i-operator ρ_S describing S such that $\rho_S \otimes \rho_T$ describes the composite system $S + T$.*

In Postulate 7, ρ_S is called an i-operator corresponding to ρ_T.

7. Condensation of Composite System

Consider a composite system $S + T$ consisting of systems S and T. Let the system T be of a large number of degrees of freedom, and is condensed with a sufficiently long

period of condensation if it is isolated. Let systems S and T interact with each other only in the period of $t_1 < t < t_2$ so that the condensation is dissolved in the period.

For $t < t_1$, system T does not interact with system S. Due to the continuity of i-operators, the composite system $S + T$ is described by a separated i-operator as

$$\rho_{S+T}(t_1) = \rho_S(t_1) \otimes \rho_T(t_1). \tag{14}$$

For $t_1 < t < t_2$, where the condensation is dissolved, the system $S + T$ develops by the unitary operator $U_{S+T}(t_1, t_2)$ and is described by

$$\rho_{S+T}(t_2) = U_{S+T}(t_1, t_2)\rho_{S+T}(t_1)U^\dagger_{S+T}(t_1, t_2). \tag{15}$$

The Hamiltonian H_{S+T} defined by $U_{S+T}(t_1, t_2) = \exp\{-i(t_2 - t_1)H_{S+T}/\hbar\}$ is supposed to be of the following form:

$$H_{S+T} = H_S + H_T + H_{\rm int}, \tag{16}$$

where H_S and H_T are Hamiltonians of systems S and T, respectively, and $H_{\rm int}$ is an interaction term. $H_{\rm int}$ depends on time such that it vanishes for $t < t_1$ and $t > t_2$.

Since system T is condensed again for $t > t_2$, there exists a set $\{\rho^m_T(t_2)\}$ of i-operators for each subspace of condensation such that $\rho_{S+T}(t_2)$ is expanded as

$$\rho_{S+T}(t_2) = \sum_m p^m \rho^m_S(t_2) \otimes \rho^m_T(t_2), \tag{17}$$

where $p^m\rho^m_S(t_2)$ is the operator-valued expansion coefficient for $\rho^m_T(t_2)$. The factor p^m in $p^m\rho^m_S(t_2)$ is determined as the remnant for $\rho^m_S(t_2)$ whose trace is unity. Due to Postulate 4, (17) means that the composite system $S+T$ is described by $\rho^m_S(t_2)\otimes\rho^m_T(t_2)$ with probability p^m. Hence all the factors $\{p^m\}$ are positive. Intuitively speaking, the relation between i-operators $\rho^m_S(t_2)$ and $\rho^m_T(t_2)$ is as follows: when system T is confined in a subspace of condensation, system S which interacts with system T is forced in a restricted region of the Hilbert space of S.

If, in addition to the i-operator $\rho_{S+T}(t_2)$, we have the information that the label of the system T is m, then we can describe the composite system $S+T$ by $\rho^m_S(t_2)\otimes\rho^m_T(t_2)$. Since system S is isolated for $t > t_2$, the i-operator

$$\rho^m_S(t_2) \tag{18}$$

describes the system S. The same argument stands for any label m which is given as information. If we consider system S and do not include the information of system T having label m, then the i-operator describing system S is

$$\tilde{\rho}_S(t_2) \equiv \sum_m p^m \rho^m_S(t_2). \tag{19}$$

By using the time-development unitary operator $U_S(t_1, t_2) \equiv \exp\{-iH_S(t_2 - t_1)\}$, we define i-operator $\rho^m_S(t_1)$ as

$$\rho^m_S(t_2) = U_S(t_1, t_2)\rho^m_S(t_1)U^\dagger_S(t_1, t_2). \tag{20}$$

That is, $\rho^m_S(t_1)$ is an i-operator which is developed in reverse time from $\rho^m_S(t_2)$ when system S does not interact with system T. By the same unitary operator, i-operator

(19) develops in reverse time to the i-operator

$$\tilde{\rho}_S(t_1) \equiv \sum_m p^m \rho_S^m(t_1). \tag{21}$$

The i-operator $\tilde{\rho}_S(t_1)$ is not the same as $\rho_S(t_1)$ in (14), but it describes the system S when we do not include the information that system S interacts with system T. We usually know only the i-operator $\tilde{\rho}_S(t_1)$ from experiment or calculation at t_2, and not $\rho_S(t_1)$ since the time-development calculation for the composite system $S+T$ by using $U_{S+T}(t_1, t_2)$ is very difficult.

Returning to (19), it means that system T determines a set $\{\rho_S^m(t_2)\}$ of i-operators for system S. For example, we consider a single spin $\boldsymbol{s} = (s_x, s_y, s_z)$ with magnitude $\frac{1}{2}$ as system S. If system T gives $\{|\uparrow\rangle\langle\uparrow|, |\downarrow\rangle\langle\downarrow|\}$ as $\{\rho_S^m(t_2)\}$, then it distinguishes spin orientations in the z-direction, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenvectors of s_z. On the other hand, if system T gives $\{|\rightarrow\rangle\langle\rightarrow|, |\leftarrow\rangle\langle\leftarrow|\}$, then it distinguishes spin orientations in the x-direction, where $|\rightarrow\rangle$ and $|\leftarrow\rangle$ are eigenvectors of s_x. Thus, the system T, which is condensed by system S as a trigger, has a specified quantization axis. In contrast, the system S does not distinguish between the quantization axes; e. g., $\frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)$ and $\frac{1}{2}(|\rightarrow\rangle\langle\rightarrow| + |\leftarrow\rangle\langle\leftarrow|)$ are completely the same.

The set $\{p^m\}$ of probabilities in (19) is not determined just by system T. We argue it in the next section of measurement.

8. Measurement

Based on the preceding section, we define the *measurement* as follows:

Definition 6 (Measurement): *Let systems S and T form a composite system, where both interact with each other only in a finite period. The system T is condensed with a sufficiently large period of condensation except for the interaction period, and the condensation is dissolved only in the interaction period. Then, we call the system S the object of measurement, the system T the apparatus of measurement, and the composite system $S+T$ the measurement system. We say that the object S of measurement is measured by the apparatus T of measurement. After the measurement, if we describe the apparatus T of measurement by an i-operator in a subspace of condensation with label m, then we call m the value of the scale in the apparatus T of measurement.*

A measurement is a phenomenon between two systems which interact with each other. The phenomenon is completely objective irrespective of the existence or the consciousness of any observer. If such a phenomenon naturally takes place without human concern, we also refer it as a measurement. Therefore, there is no additional postulate to explain measurements.

The system $S+T$ in the preceding section is a measurement system. The system S at t_1 is described by $\rho_S(t_1)$ as seen in (14). Then the system S interacts with the system T in the period of $t_1 < t < t_2$, and we have the value m of the scale in the apparatus T of measurement. Hence we can describe the system S at t_2 by $\rho_S^m(t_2)$ in (18). In the

measurement process, $\rho_S(t_1)$ does not become $\rho_S^m(t_2)$ by any unitary transformation. Although the change from $\rho_S(t_1)$ to $\rho_S^m(t_2)$ corresponds to a collapse of a wave function in the conventional quantum mechanics, it is not a jump in the time development. We just selected an i-operator among multiple i-operators describing the system S for our purpose after the measurement. In fact we may select $\rho_{S+T}(t_2)$ in (15) to describe the composite system $S+T$ at t_2. However this i-operator involves quantities of T as well as those of S, so that it is difficult to extract useful information about the system S. A typical observer needs the value of the scale and its probability in the apparatus T. And, after the measurement, he needs an i-operator describing only the object S of measurement. Then, if he thinks reasonably, he describes the system by ρ_S before measurement, and describes it by ρ_S^m after measurement if he has the information that the value of the scale is m. Since both ρ_S and ρ_S^m are separately continuous with respect to time, the measurement introduce no discontinuity.

We now determine the set $\{p^m\}$ of probabilities in $\tilde{\rho}_S(t_2)$ of (19), which describes the system S after the measurement. To be precise, we adopt $\{p^m\}$ rather than determine it. At first, we consider the case that we know the i-operator (14) describing the measurement system $S+T$ at t_1. Also let it be possible to calculate (15) by the time-development unitary operator $U_{S+T}(t_1, t_2)$. Further let it be possible to expand the obtained i-operator in the form of (17). In this case, we may adopt $\{p^m\}$, which has been determined in the expansion coefficients, as a set of probabilities. This i-operator $\rho_{S+T}(t_2)$ describes the composite system at t_2 in the case that we know $\rho_{S+T}(t_1)$ as information before the measurement. However, it is not necessarily possible to know $\rho_{S+T}(t_1)$ or to calculate $\rho_{S+T}(t_2)$.

Another way of adopting $\{p^m\}$ is found, if we prepare N equivalent measurement systems. The number N need not but may be large. We perform the equivalent measurement for each measurement system. Then, as p^m for each value m of the scale, we adopt the ratio of the obtained number of the value m against the total number N of the experiments. This choice of $\{p^m\}$ determines an i-operator (19) which describes the system S. This is the i-operator which includes the information from the specific N experiments.

We now perform another set of equivalent N experiments. Then we have other values for $\{p^m\}$, since the number obtaining the value m is generally different from the previous. We hence adopt another set of values for $\{p^m\}$. Thus the system S is described by a different i-operator for each set of experiments. There is no contradiction between these descriptions, since each description just includes different information about the system S. As an extreme case, we can specify arbitrary values for $\{p^m\}$ satisfying $\sum_m p^m = 1$ without performing experiments. Then we can say that (19) with the values for $\{p^m\}$ also describes the system S. However the i-operator does not include any information from any experiments and hence is not useful. Thus it is important not only that the i-operator describes the system but also that it includes information which we need.

We compare the results, when we repeat many sets of N experiments. Although

the sets of the obtained values for $\{p^m\}$ are generally different from each other, we may usually have the following physical expectation. That is, many sets of the values for $\{p^m\}$ may be close to each other. As a useful choice in the case, we adopt the i-operator (19) with $\{p^m\}$ where each p^m is the average for the sets of experiments. On the other hand, we do not exclude the possibility that the values for $\{p^m\}$ depend strongly on each set of experiments. In such cases, we do not predict the results for the next set of experiments.

From (19), $\tilde{\rho}_S(t_2)$ is a common expansion of $\{\rho_S^m(t_2)\}$ from Theorem 3. We denote the contracting operator for each m as K_S^m; i. e. $\rho_S^m(t_2) = K_S^m \tilde{\rho}_S(t_2)(K_S^m)^\dagger$. Then we have the following equation:

$$\tilde{\rho}_S(t_2) = \sum_m p^m K_S^m \tilde{\rho}_S(t_2)(K_S^m)^\dagger. \tag{22}$$

By defining $M_S^m \equiv \sqrt{p_T^m} K_S^m$ $(m = 1, 2, \cdots)$, this equation becomes

$$\tilde{\rho}_S(t_2) = \sum_m M_S^m \tilde{\rho}_S(t_2)(M_S^m)^\dagger. \tag{23}$$

Then we have

$$p^m = \mathrm{tr}\{M_S^m \tilde{\rho}_S(t_2)(M_S^m)^\dagger\}. \tag{24}$$

In a general measurement system, $\{M_S^m\}$ in (23) depends on $\tilde{\rho}_S(t_2)$. We define a special class of measurement systems as follows:

Definition 7 (Definitive Measurement): *A measurement system $S+T$ where* (23) *is an identity is a definitive measurement system.*

That is, $\{M_S^m\}$ is determined independently of each $\tilde{\rho}_S(t_2)$ in a definitive measurement system. By taking a trace of (23), we have $\mathrm{tr}\{\tilde{\rho}_S(t_2)\sum_m (M_S^m)^\dagger M_S^m\}$ $= 1$. Since this stands for any $\rho_S(t_2)$ in the definitive measurement system, we have

$$\sum_m (M_S^m)^\dagger M_S^m = I_S. \tag{25}$$

By using $\{M_S^m\}$, the observable is defined as follows:

Definition 8 (Observable): *In a definitive measurement system $S+T$, Hermitian operator*

$$F = \sum_m f(m)(M_S^m)^\dagger M_S^m \tag{26}$$

with a real function $f(m)$ is an observable of the system S for the apparatus T of measurement.

Although M_S^m is an operator in the Hilbert space $\mathcal{H}_S$, the definition depends on the condensation of the system T. In general, $f(m)$ is not an eigenvalue of F. In the special case that M_S^m is the projection operator for all m, the definition of F is the same as that of an observable in the conventional quantum mechanics and $f(m)$ becomes an eigenvalue of F.

By (24) and (26), the expectation value of the observable F is written as

$$\bar{F} = \sum_m f(m)\, p^m = \mathrm{tr}\{F\rho_S(t_2)\}. \tag{27}$$

This reads as follows: When observable F of the object S of measurement is measured by the apparatus T of measurement, the probability that the value of the scale is m is p^m. Then the value of observable F is $f(m)$. If we repeat N times the same measurement to N equivalent systems, then we physically expect that the average of the obtained values of $f(m)$ reaches $\bar{F}$ with increasing N.

9. Information Vector

As has been defined, an i-operator ρ is pure if it is written as

$$\rho = |\psi\rangle\langle\psi| \tag{28}$$

with a vector $|\psi\rangle$ in the Hilbert space of the system. This expression is not unique, since we can use

$$|\psi'\rangle = e^{i\alpha}|\psi\rangle \tag{29}$$

with arbitrary real number α and write it as

$$\rho = |\psi'\rangle\langle\psi'|. \tag{30}$$

For a pure i-operator, $\rho = |\psi\rangle\langle\psi|$, we can identify it by specifying the vector $|\psi\rangle$, or $|\psi'\rangle = e^{i\alpha}|\psi\rangle$. Hence it is allowed to use the vector $|\psi\rangle$ as an agent for ρ. In this usage, we call $|\psi\rangle$ the information vector. Thus an arbitrariness of a phase factor is brought out, although it is not in the original i-operator. The information vector $|\psi\rangle$ is only an agent for the corresponding i-operator, and does not represent a *state* of the system in the conventional quantum mechanics. Even if a pure i-operator describes a system, another i-operator which is not pure may describe the same system simultaneously.

On the other hand, we can treat an information vector as if it is a state vector in the conventional quantum mechanics. In particular, we have a new information vector by superposing two information vectors. An information vector describing the system follows the Schrödinger equation. The probability interpretation stands like the conventional quantum mechanics. Therefore the information vector covers all the same description region of the state vector in the conventional quantum mechanics.

10. Stern-Gerlach Experiment

We consider the Stern-Gerlach experiment with an apparatus in a standard arrangement. Two magnets are placed in a way that the north pole of a magnet and the south pole of the other magnet face each other in the z-direction. The shape of one magnet is acute and that of the other is plane so as to yield an inhomogeneous magnetic field. Let silver atoms be ejected and let them travel through the magnetic field. Behind the magnets a screen is placed to stop the silver atoms. A silver atom has a spin $\boldsymbol{s}$ of

magnitude $s = \frac{1}{2}$ which comes from the outermost electron. The ejected atoms are very dilute so that the problem is of a single atom. After passing through the magnetic field, the atom is deflected positively or negatively in the z-direction and makes a spot on one of two places of the screen. This is because the atom is attracted to the negative direction if the z-component of the spin is $+\frac{1}{2}$ and to the positive direction if it is $-\frac{1}{2}$ in the inhomogeneous magnetic field. The deflection becomes opposite depending on the arrangement and the forms of the magnets. We examine this phenomenon in terms of i-operators as follows.

When a silver atom is ejected, it is not affected by the distant magnets. The atom reaches and passes through the magnet, so that it interacts with the magnets only in a finite period $t_1 < t < t_2$. After then, the atom is isolated from the magnets again. When the atom does not interact with the magnets, the spin degree of freedom is also independent of the orbital degree of freedom in the atom. At $t = t_1$, let $\rho_{\rm mag}(t_1)$ be an i-operator describing the magnets, $\rho_{\rm orb}(t_1)$ be an i-operator describing the orbital degree of freedom, $\rho_S(t_1)$ be an i-operator describing the spin degree of freedom. Then the total system is described by the i-operator $\rho_S(t_1) \otimes \rho_{\rm orb}(t_1) \otimes \rho_{\rm mag}(t_1)$. As for the spin degree of freedom, we denote the eigenvector for eigenvalue $+\frac{1}{2}$ of s_z as $|\uparrow\rangle$, and the eigenvector for $-\frac{1}{2}$ as $|\downarrow\rangle$. Then the corresponding pure i-operators are $\rho_S^{\uparrow} \equiv |\uparrow\rangle\langle\uparrow|$ and $\rho_S^{\downarrow} \equiv |\downarrow\rangle\langle\downarrow|$, respectively. Since we have no information about the spin direction of the ejected atom, we describe the spin by the i-operator $\rho_S(t_1) = \frac{1}{2}(\rho_S^{\uparrow} + \rho_S^{\downarrow})$.

Here we regard the total system as a composite system of systems S and T, where the system S is only of the spin degree of freedom in the atom, and the system T is further a composite system of the magnets and the orbital degree of freedom of the atom. The latter is described by the i-operator $\rho_T(t_1) \equiv \rho_{\rm orb}(t_1) \otimes \rho_{\rm mag}(t_1)$. Then the total system at $t = t_1$ is described by i-operator $\rho_{S+T}(t_1) = \rho_S(t_1) \otimes \rho_T(t_1)$, which is of the same form as (14).

The system T is condensed for $t < t_1$ and $t > t_2$, in which periods it does not interact with the system S. Actually, since the magnets are macroscopic, it is supposed that the weight or probability for the part with each value of the momentum of the magnets does not change with time in a total i-operator describing the system T. The orbital degree of freedom of the atom does not affect the condensation of the magnets and the system T is also condensed. Here we define that the value m of the scale of measurement takes -1, 0 or 1 according to the sign of the z-component of the momentum of the system T. For $t < t_1$, the magnets stands still and then the system T is condensed with $m = 0$. For $t_1 < t < t_2$, the system T interacts with the system S, so that the condensation is dissolved and then the system T obtains a different value of the momentum. Since the momentum difference is quite small, we cannot detect it in reality. However, since the total momentum of the magnets and the atom is conserved, the change of the momentum of magnets is reflected in the atomic orbital for $t > t_2$. Therefore we read the value m as -1 for an orbital deflected above, 0 for an straight orbital, and 1 for an orbital deflected below.

We consider phenomenologically the time development of the system $S + T$ for

$t_1 < t < t_2$, in which period the system is dissolved from the condensation. Typical i-operators for the system T are $\rho_T^0 = |\psi_T^0\rangle\langle\psi_T^0|$, $\rho_T^+ = |\psi_T^+\rangle\langle\psi_T^+|$ and $\rho_T^- = |\psi_T^-\rangle\langle\psi_T^-|$: ρ_T^0 represents the motion that the atomic orbital is straight and the z-component of the momentum of the magnets is zero; ρ_T^+ (ρ_T^-) represents the motion that the atomic orbital is deflected upward (downward) and the z-component of the momentum of the magnets is negative (positive). Here we define a pseudo spin $\boldsymbol{R} = (R_x, R_y, R_z)$ of magnitude unity such that $|\psi_T^-\rangle$, $|\psi_T^0\rangle$ and $|\psi_T^+\rangle$ are eigenvectors belonging to eigenvalues, -1, 0 and 1, respectively, of R_z.

The time development of an i-operator in the period of $t_1 < t < t_2$ is given by the Hamiltonian H for the system $S+T$ or by the unitary operator $U(t_1, t_2) = \exp(-iH(t_2 - t_1)/\hbar)$. Here we express this time development by the following phenomenological unitary operator U instead of $U(t_1, t_2)$:

$$U = \frac{1}{4}[(1 - 2s_z) \otimes \tilde{R}_+ + (1 + 2s_z) \otimes \tilde{R}_-], \tag{31}$$

with

$$\tilde{R}_\pm \equiv \sqrt{2}(R_z \pm 1)R_x(R_z \pm 1) + R_z(R_z \mp 1). \tag{32}$$

Since the atomic orbital is straight before the interaction, the system $S+T$ at $t = t_1$ is described by i-operator

$$\rho_{S+T}(t_1) = \frac{1}{2}(\rho_S^\uparrow + \rho_S^\downarrow) \otimes \rho_T^0. \tag{33}$$

By straightforward calculation, the i-operator develops to the following i-operator at $t = t_2$:

$$\rho_{S+T}(t_2) = U\rho_{S+T}(t_1)U^\dagger = \frac{1}{2}(\rho_S^\uparrow \otimes \rho_T^- + \rho_S^\downarrow \otimes \rho_T^+). \tag{34}$$

Hence the system $S + T$ at $t = t_2$ is described by $\rho_{S+T}(t_2)$. This i-operator means that the system is described by $\rho_S^\uparrow \otimes \rho_T^-$ with probability $\frac{1}{2}$ and by $\rho_S^\downarrow \otimes \rho_T^+$ with probability $\frac{1}{2}$. For $t > t_2$, since the system is condensed again, $\rho_{S+T}(t_2)$ keeps the same form as (34): i. e. $\rho_T^-(t)$ ($\rho_T^+(t)$) develops within the subspace of $m = -$ ($m = +$) without mixing.

We suppose to have the information that the atom reached the screen and a spot appeared at a downward deflected position. It means that we have $m = -$ for the value of the scale in the system T. The probability that this occurs is $\frac{1}{2}$ from (34). If we include this information, we describe the total system by $\rho_S^\uparrow \otimes \rho_T^-$. Owing to the separated form of the i-operator, the system S is described by i-operator $\rho_S^\uparrow$. Similarly, we suppose to have the information that the atom reached the screen and a spot appeared at an upward deflected position, and we suppose to include the information. Then we describe the system S by i-operator $\rho_S^\downarrow$.

The conventional quantum mechanics may explain the situation that the value of the scale is $m = -$ as follows: the density operator (34) discontinuously changes into $\rho_S^\uparrow \otimes \rho_T^-$. In contrast, the present reconstructed quantum mechanics produces no discontinuity in any i-operators. For $t > t_2$, the total system is described by the i-operator (34) as well as $\rho_S^\uparrow \otimes \rho_T^-$. These two i-operators are continuous for all period

including $t \leq t_2$. An observer usually prefers the information of $m = -$, and describes the system by $\rho_S^{\uparrow} \otimes \rho_T^{-}$. On the other hand, an observer may not care the value m, or may not have the information of $m = -$. Then the observer may describe the same system by i-operator (34). If we pay attention only to the system S, we say the followings: The system S is described by $\rho_S^{\uparrow}$ if the information of $m = -$ is included, and by $\frac{1}{2}(\rho_S^{\uparrow} + \rho_S^{\downarrow})$ if the information is not included.

11. Electron through Screen with Slits

We examine a slit experiment where an electron is ejected to a solid thin screen that has typically two slits cut into it. Behind the slit screen, another detection screen is also set up to record what comes. This is performed as a real experiment [2], and is regarded as an interference experiment of an electronic wave function in the conventional quantum mechanics. Hereafter we consider only the orbital degree of freedom, which affects experimental results, and do not pay attention to the spin degree of freedom.

The electron reached the slit screen is absorbed or reflected by the material of the slit screen with a large probability, and cannot go through it. Only in a small probability, say p, the electron reaches the detection screen. Then the electron interacts with the slit screen for $t_a < t < t_b$. Thus the electron from the ejection time to the time reached to the slit screen is supposed to be described by the i-operator

$$\rho(t) = p\rho_a(t) + (1-p)\rho_{\rm abs}(t), \quad (t < t_a) \tag{35}$$

where $\rho_a(t) \equiv |\psi_a(t)\rangle\langle\psi_a(t)|$ is an i-operator representing the passage, and $\rho_{\rm abs}(t)$ is an i-operator representing the non-passage. Then p is the probability that the electron passes through the slits. By denoting an i-operator describing the slit screen as $\rho_{\rm slit}(t)$, the total system is described by a separated i-operator $\rho(t) \otimes \rho_{\rm slit}(t)$ $(t < t_a)$. After t_a, the electron interacts with the slit screen, so that the i-operator develops with time into an unseparated form.

Now we suppose to be interested in phenomena for $t > t_b$ only when the electron passes through the slit screen. In this case, it is not convenient to use the i-operator which is time-developed from $\rho(t)$ in (35). In fact $\rho(t)$ is not separated into an electronic factor and a slit screen factor, and further it even describes the unnecessary possibility that the electron is absorbed in the slit screen to vanish. We have examined only the case that the electron passes through the slit screen, and the passage is judged by a spot on the detection screen. In the case that we found a spot, the electron clearly passed through the slit screen. Hence it is reasonable to describe the electron by a pure i-operator

$$\rho_b(t) = |\psi_b(t)\rangle\langle\psi_b(t)|. \quad (t > t_b) \tag{36}$$

The total system is then described by an i-operator in the form of $\rho_b(t) \otimes \rho'_{\rm slit}(t)$.

Here we assume that $\rho_a(t)$, a part of (35), continues to $\rho_b(t)$ in (36) with time. That

is, we assume the existence of a pure i-operator $\rho_{\rm int}(t) \equiv |\psi_{\rm int}(t)\rangle\langle\psi_{\rm int}(t)|$ such that

$$\tilde{\rho}(t) = \begin{cases} \rho_a(t) & (t < t_a) \\ \rho_{\rm int}(t) & (t_a < t < t_b) \\ \rho_b(t) & (t > t_b) \end{cases} \tag{37}$$

is continuous with time. This assumption seems to be physically allowable, although it is not trivially guaranteed. In terms of information vectors, we have assumed that there exists a continuous vector $|\psi(t)\rangle$ such that $|\psi(t)\rangle = |\psi_a(t)\rangle$ $(t < t_a)$, $|\psi(t)\rangle = |\psi_{\rm int}(t)\rangle$ $(t_a < t < t_b)$ and $|\psi(t)\rangle = |\psi_b(t)\rangle$ $(t > t_b)$, if the phase factor is appropriately chosen. The interference effect for the *wave function* of the electron in the conventional quantum mechanics is actually for the information vector $|\psi(t)\rangle$.

12. Summary

We have reconstructed quantum mechanics based on two central concepts. One of them is the multiple description of a physical system. There are multiple inequivalent i-operators to simultaneously describe a single system, and the different i-operators carry different kinds or amounts of information for the same system. There is no preferential i-operator which plays a specially important role. What we can do is to draw information out from the i-operator which we have. Accordingly we discarded the concept of the state, or the wave function, which has played the central role in the conventional quantum mechanics.

The other concept in the reconstructed quantum mechanics is the condensation of a system. For a condensed system, the Hilbert space decomposes into subspaces where the time-development unitary operator or the Hamiltonian cannot overcome the boundaries of the subspaces. Reading the label of the subspace, we can adopt an i-operator belonging to the subspace to describe the system.

The measurement is defined as the phenomenon between two systems S and T where T is condensed if it is isolated, S and T interact with each other in a finite period, and the condensation of T is dissolved in the interaction period. In the measurement system $S+T$, we have called T the apparatus of measurement and S the object of measurement. The results of the measurement is the label of the subspace of condensation and the i-operator belonging to the subspace. We have called the label the value of the scale of measurement. By the concepts of the multiple description of a system by the i-operators and the concept of the condensation of the system, we can explain quantum phenomena including the measurement without any discontinuity. Especially we need no postulate specific to the measurement.

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